Exercises for AO2 Trig and log Proofs

1 By putting $u = \log_4 r$, show that $\log_4 r = \frac{1}{2}\log_2 r$

A $u = \log_4 r$ so $r = 4^u$

$r = 4^u$

$r = 2^{2u}$

$r^{\frac{1}{2}} = 2^u$

$\log_2 r^{\frac{1}{2}} = u$

$\frac{1}{2}\log_2 r = \log_4 r$

B $\log_4 r = \log_{2^2} r$

$\log_4 r = \log_2 \sqrt{r}$

$\log_4 r = \frac{1}{2}\log_2 r$

C $u = \log_4 r$ so $r = 4^u$

$r = 4^u = 2 \times 2^u$

$\frac{r}{2} = 2^u$

$\frac{1}{2}\log_2 r = u$

$\frac{1}{2}\log_2 r = u$

D $\log_4 r = \frac{\ln r}{\ln 4}$

$\frac{\ln r}{\ln 4} = \frac{\ln r}{2\ln 2}$

$\frac{\ln r}{2\ln 2} = \frac{1}{2}\log_2 r$

E $\frac{1}{2}\log_2 r = \log_4 r$

$\frac{\log_2 r}{\log_2 4} = \log_4 r$

$\log_4 r = \log_4 r$

Exercises for AO2 Trig and log Proofs

2. Prove that $\frac{1+\sin 2x}{\cos 2x} = \frac{1+\tan x}{1-\tan x}$

A

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+2\sin x\cos x}{\cos^2 x-\sin^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{\cos^2 x+\sin^2 x+2\sin x\cos x}{\cos^2 x-\sin^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{(\cos x+\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x+\sin x}{\cos x-\sin x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x+\sin x}{\cos x-\sin x} \div \frac{\cos x}{\cos x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+\tan x}{1-\tan x}$$

B

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+2\sin x\cos x}{\cos^2 x-\sin^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+2\sin x\cos x}{\cos^2 x-\sin^2 x} \div \frac{\cos^2 x}{\cos^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{\sec^2 x+2\tan x}{1-\tan^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+\tan^2 x+2\tan x}{1-\tan^2 x}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{(1+\tan x)^2}{(1+\tan x)(1-\tan x)}$$

$$\frac{1+\sin 2x}{\cos 2x} = \frac{1+\tan x}{1-\tan x}$$

Exercises for AO2 Trig and log Proofs

C $\frac{1+\tan x}{1-\tan x} = \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}$

$$\frac{1+\tan x}{1-\tan x} = \frac{\cos x+\sin x}{\cos x-\sin x}$$

$$\frac{1+\tan x}{1-\tan x} = \frac{(\cos x+\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)}$$

$$\frac{1+\tan x}{1-\tan x} = \frac{\cos^2 x+\sin^2 x+2\sin x\cos x}{\cos^2 x-\sin^2 x}$$

$$\frac{1+\tan x}{1-\tan x} = \frac{1+2\sin x\cos x}{\cos^2 x-\sin^2 x}$$

$$\frac{1+\tan x}{1-\tan x} = \frac{1+\sin 2x}{\cos 2x}$$

D $\frac{1+\sin 2x}{\cos 2x} = \frac{1+\tan x}{1-\tan x}$

$$(1+\sin 2x)(1-\tan x) = \cos 2x(1+\tan x)$$

$$1+\sin 2x-\tan x-\sin 2x\tan x = \cos 2x\tan x$$

$$\cos x+\cos x\sin 2x-\sin x-\sin 2x\sin x = \cos 2x\cos x+\cos 2x\sin x$$

$$\cos x+\cos x\sin 2x-\sin x-\cos 2x\sin x = \cos 2x\cos x+\sin 2x\sin x$$

$$\cos x-\sin x = \cos x-\sin x$$